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# The lateral distribution of muons in near vertical EAS

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**Abstract.** A large-area muon detector ($12{\cdot}5\ m^2$) has been used to investigate the muon component of large EAS at Haverah Park of size $0{\cdot}27 < \rho_{500} < 2{\cdot}5$ for zenith angle ($\theta$) < 25°. The analysis is based on 821 showers covering core distances of 160 m to 500 m. The results are compared with other observations and with recent model calculations. It is found that only certain models are in reasonable agreement with the experimental data.

## 1. Introduction

The muon detector is situated at the centre of the Haverah Park array and has been described in detail by Blake *et al* (1970, 1971). The detector, which is shown in figure 1, consists of three separate units of equal area. Each unit is a multilayer sandwich of neon flash tubes and lead absorber. The flash tubes (2 m × 1·8 cm) are arranged in four double layers each separated by 5 cm of lead and 2·5 cm of supporting steel. A determination of the number of muons ($E > 300 \sec\theta$ MeV) and electrons striking the detector can thus be carried out. The flash-tube units and recording cameras are triggered by the main Haverah Park 500 m array which detects EAS produced by primaries of $10^{17}$ eV and above.

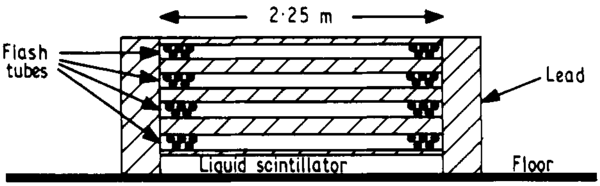


**Figure 1.** The muon detector.

The main 500 m array at Haverah Park consists of water Čerenkov detectors of depth 1·2 m. The $\rho_{500}$ value assigned to a particular EAS is the estimated energy loss density of the shower in traversing a uniform slab of water 1·2 m thick at a distance of 500 m from its axis.

The approximate relationship between the primary energy and $\rho_{500}$ given by Hillas *et al* (1971) is

$$E_p \simeq 4 \times 10^{17} (\rho_{500})^{1{\cdot}02}\ \mathrm{eV}.$$

($\rho_{500}$ is expressed as vertically equivalent muons per square metre (ve$\mu$m$^{-2}$).)

## 2. Selection and analysis of data

In the previous Nottingham work (Blake *et al* 1971) most showers triggering the Haverah Park array for which $\theta < 25°$, were used in the muon analysis. This paper reports the results of a further year's observation and includes a more restrictive choice of the data used; only those showers whose core location and shower size are most rigidly determined are included. In order that a shower is accepted for analysis the criteria are adopted that:

(i) the density at three of the detectors of the Haverah Park 500 m array must exceed 0·45 ve$\mu$m$^{-2}$, that is, one-and-a-half times the electronic triggering levels; and

(ii) showers must fall within $150\,\text{m} < R < 500\,\text{m}$ of the muon detector and at distances greater than 150 m from any Čerenkov detector.

These more stringent selection criteria reduce the possibility of error due to local fluctuations in the triggering of the array (Evans 1971). The above criteria were satisfied by 821 showers of size $0{\cdot}27 < \rho_{500} < 2{\cdot}5$ and zenith angle $\theta < 25°$.

In order to obtain an accurate value for the muon density the overall efficiency and spatial resolution of the detector are taken into account (Blake *et al* 1971).

## 3. The muon lateral density distribution

For six mean values of $\rho_{500}$ the mean density of muons at several core distances ($R$) was obtained. Typical examples of this data are shown in figure 2, and the complete data is given in table 1.

Previously (see Blake *et al* 1971) a function of the form

$$\rho_\mu = kR^{-n} \tag{1}$$

where $k$ is a constant, was fitted to the data. A mean value of $n = 2{\cdot}2 \pm 0{\cdot}07$ was obtained

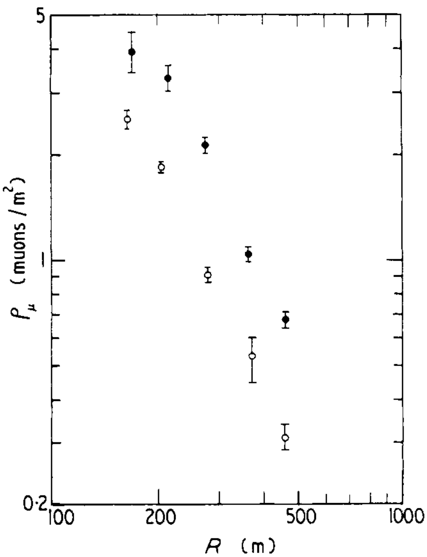


**Figure 2.** Density of muons as a function of core distance.

**Table 1.** Mean density ($m^{-2}$) of muons in $\rho_{500}$ (ve$\mu m^{-2}$) and $R$ (m) bins

| Mean $\rho_{500}$ | Mean | | | | | |
|---|---|---|---|---|---|---|
| 0·277 | $R$(m) | 174 | 203 | 256 | 348 | |
| | $\rho_\mu$ | 1·35 ± 0·35 | 0·92 ± 0·11 | 0·60 ± 0·08 | 0·45 ± 0·12 | |
| 0·401 | $R$(m) | 166 | 211 | 282 | 358 | 451 |
| | $\rho_\mu$ | 1·97 ± 0·16 | 1·16 ± 0·06 | 0·59 ± 0·04 | 0·44 ± 0·03 | 0·45 ± 0·09 |
| 0·564 | $R$(m) | 164 | 205 | 280 | 371 | 460 |
| | $\rho_\mu$ | 2·52 ± 0·13 | 1·83 ± 0·07 | 0·91 ± 0·04 | 0·53 ± 0·08 | 0·31 ± 0·03 |
| 0·800 | $R$(m) | 164 | 208 | 276 | 370 | 460 |
| | $\rho_\mu$ | 3·44 ± 0·24 | 2·39 ± 0·12 | 1·38 ± 0·06 | 0·61 ± 0·03 | 0·43 ± 0·03 |
| 1·210 | $R$(m) | 170 | 215 | 273 | 372 | 464 |
| | $\rho_\mu$ | 3·94 ± 0·59 | 3·31 ± 0·26 | 2·13 ± 0·08 | 1·03 ± 0·05 | 0·68 ± 0·05 |
| 1·795 | $R$(m) | | 197 | 280 | 382 | 459 |
| | $\rho_\mu$ | | 4·12 ± 0·53 | 2·59 ± 0·13 | 1·57 ± 0·09 | 0·86 ± 0·11 |

(Ferguson 1971). Fitting a function of this form to the present data gave a mean value of $n = 2{\cdot}07 \pm 0{\cdot}10$ which indicated that the present analysis using only the best located showers has made no significant difference to the value of $n$ obtained.

The work of Andrews *et al* (1971) and McCusker *et al* (private communication) show different forms of the density structure function. Andrews *et al* use a function, first proposed by Linsley (1964), of the form

$$\rho_{\mathrm{Ch}} \propto \frac{1}{R}\left(1+\frac{R}{R_0}\right)^{-(\eta-1)} \tag{2}$$

where $R_0 = 600 \sec\theta - 400$ and $\eta = 4{\cdot}0 - 0{\cdot}5 \sec\theta$, for the Čerenkov tank response.

McCusker *et al* use a function of the form

$$\rho_\mu(N_\mu, R_1) = \left(\frac{N_\mu}{4{\cdot}5\times10^6}\right)\frac{25}{\{1+(R/300)\}^3}\,\mathrm{e}^{-R/1000}$$

where $N_\mu$ is the 'scintillator' muon size for $0° < \theta < 40°$.

A detailed analysis using a structure function of similar form to equation (2) gives a slightly better fit to the data than a simple power law on the basis of $\chi^2$ tests. (Both functions (1) and (2) give high degrees of significance.) This structure function can be expressed as

$$\rho_\mu \propto \frac{1}{R}\left(1+\frac{R}{276}\right)^{-2\cdot1}.$$

Both values of $\eta$ and $R_0$ ($3{\cdot}1 \pm 0{\cdot}01$ and $276 \pm 5$ respectively) were minimized using the $\chi^2$ test.

The mean value of the slope, $-2{\cdot}07$, agrees well with the mean value, $-2{\cdot}1$, obtained by McCusker *et al*. However it should be realized that the threshold energies of the Nottingham and Sydney detectors are $300 \sec\theta$ MeV and 1 GeV respectively.

## 4. Comparison with model calculations

Hillas *et al* (1970) computed for several models of nuclear interactions, the muon density as a function of core distance and shower size for muons with energies in excess of the detection threshold of the present experiment, thus enabling a direct comparison to be made.

In all these calculations the following assumptions have been made:

(i) It is assumed that the inelasticities for nuceon–nucleon and pion–nucleon collisions are 0·56 and 1 respectively.

(ii) That pions produced in EAS and undergoing further interactions produce as many secondary pions as if the nucleon had radiated the same amount of energy.

(iii) All models use the same transverse momentum distribution amongst the pions, that is,

$$f(p_t) = p_t \exp\left\{-\left(\frac{P_t}{P_0}\right)\right\} \mathrm{d}p_t$$

with a mean value of $p_t = 0{\cdot}4\ \mathrm{GeV}/c$.

(iv) The interaction lengths of the pions and nucleons are assumed to be 100 and 80 g cm$^{-2}$ respectively.

The variation in the models is given in table 2. It should be mentioned that all models use a common propagation mechanism.

**Table 2.** Summary of Hillas' models

| Model | A | D | E | F | H | I | J | K |
|---|---|---|---|---|---|---|---|---|
| Fraction going into pionization | 0·44 | 0·44 | 0·44 | 0·44 | 0·44 | 0·29 | 0·29 | 0·29 |
| Fraction going into isobars | 0 | 0 | 0 | 0 | 0 | 0·15 | 0·15 | 0·15 |
| Multiplicity law | $KE^{1/4}$ | $KE^{1/4}$ | $KE^{1/4}$ | $KE^{1/4}$ | $KE^{1/4}$ $KE^{1/2}$ for $(E > 3\ \mathrm{TeV})$ | $KE^{1/4}$ | $KE^{1/4}$ $KE^{1/2}$ for $(E > 3\ \mathrm{TeV})$ | |
| Value of $K$ | Forward and backward cone multiplicity changed | 12 | 19 | 30 | 19 | 16 | 16 | 16 |
| Comments | | | | | | | Maximum number of pions from baryon decay = 4 when $E_p = 10\ \mathrm{TeV}$ | Same as I but $\frac{2}{3}$ of baryon decays neutral |

Hillas in his model calculations introduced the ratio $\rho_\mu/\rho_{\mathrm{Ch}}^{0\cdot75}$ for comparing the responses of the muon and Čerenkov detectors. The value of 0·75 is chosen so that the ratio is relatively insensitive to core-location errors. Figures 3, 4 and 5 show the experimental data for the ratio as a function of the core distance, alongside the predictions of the models. All data are for a $\rho_{500} = 0{\cdot}316$ ve$\mu$m$^{-2}$ and the shaded areas for selected models show the predicted change in the ratio caused by an increase in the primary

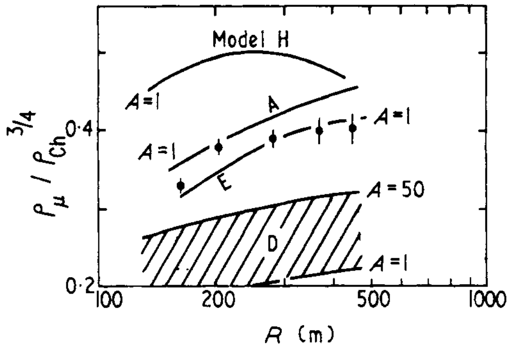


**Figure 3.** Hillas' ratio as a function of core distance.

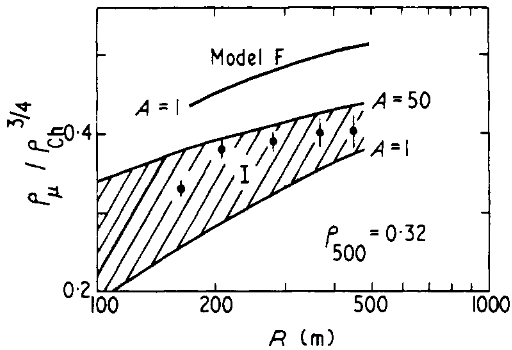


**Figure 4.** Hillas' ratio as a function of core distance.

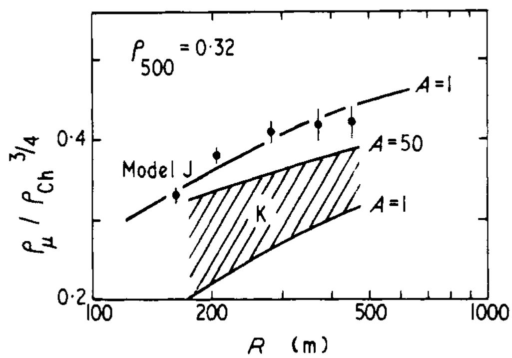


**Figure 5.** Hillas' ratio as a function of core distance.

mass from 1 to 50. The ratio depends strongly on the nuclear model and primary mass and is therefore a useful parameter.

The models which are consistent with the experimental data are model E with $A = 1$ and model K with heavy primary mass. Models I, J and A all predict a ratio which is comparable with the observed value, although all of these models predict a stronger dependence of the ratio on core distance. It is clear that models H and F must be excluded because of their high muon content.

## 5. The dependence of the muon density on the primary energy

From the previous data relating the lateral distribution to shower size, the muon density ($\rho_\mu$) at 350 m from the core was calculated for each shower size bin. The relationship between $\rho_\mu$ (350 m) and shower size ($\rho_{500}$) can be represented by an equation of the form

$$\rho_\mu(350\,\mathrm{m}) = k(\rho_{500})^\alpha.$$

Fitting this relationship to the observed data yields a best-fit value for $\alpha$, that is, $\alpha = 0{\cdot}91 \pm 0{\cdot}02$. This value may be compared with the predictions of the Hillas models which are given in table 3.

**Table 3.** Values of $\alpha$ yielded by best-fitting lines ($\pm 1\%$) to muon densities predicted by the Hillas' models

| Model | A | D | E | F | H | I | J | K |
|---|---|---|---|---|---|---|---|---|
| $A = 1$ | 0·96 | 0·92 | 0·96 | 0·98 | 0·98 | 0·95 | 0·98 | 0·95 |
| $A = 50$ | 0·96 | 0·91 | 0·96 | 0·98 | 0·98 | 0·94 | 0·98 | 0·93 |

As will be seen from the table the sensitivity of $\alpha$ to changes in primary mass is slight. The experimental observations yield a slightly lower value of $\alpha$ than any of the models predict.

The variation of $\rho_\mu/\rho_{\mathrm{Ch}}^{0\cdot75}$ with shower size for a core distance of 240 m was also studied. These results are displayed in figures 6 and 7. This time only the predictions of models D, E, I and K are shown. It is clear that model D is excluded by this analysis, but models E, I and K are possible models. The agreement with model E for proton primaries is good and with model I for $A \simeq 10$. The predictions using model K are somewhat low compared with the experimental data, but this model cannot be excluded.

The possible change in slope at $\rho_{500} = 1$, observed by Ferguson (1971) is not as pronounced in the present set of data, but the existence of the change cannot be ruled out.

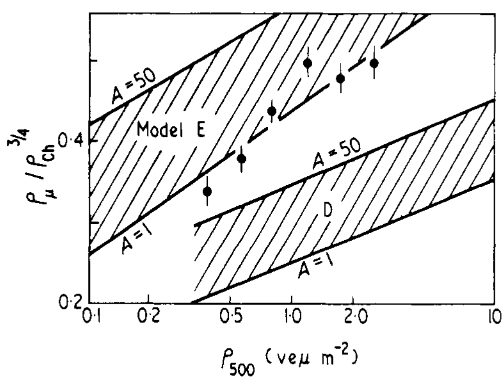


**Figure 6.** Hillas' ratio as a function of shower size.

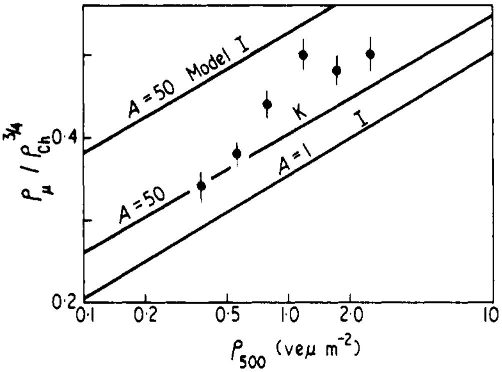


**Figure 7.** Hillas' ratio as a function of shower size.

## 6. The lateral density distribution out to 1 km

Data on showers at large core distances are recorded in association with the 2 km array at Haverah Park. This enables the muon lateral density distribution to be extended out to core distances of about 1 km (figure 8). The experimental data is from 178 showers taken from those EAS used by the Leeds group in their energy spectrum plot presented at the Hobart conference (Andrews *et al* 1971) and as such represent a sample of very accurately located showers. The data in this case are normalized to a particular shower size ($\rho_{500}$ = 2·66) by use of equation (4) with $\alpha$ = 0·91.

Also shown plotted is the prediction of the Hillas' model E with proton ($A$ = 1) primaries. This plot also closely represents the prediction of model I with primaries

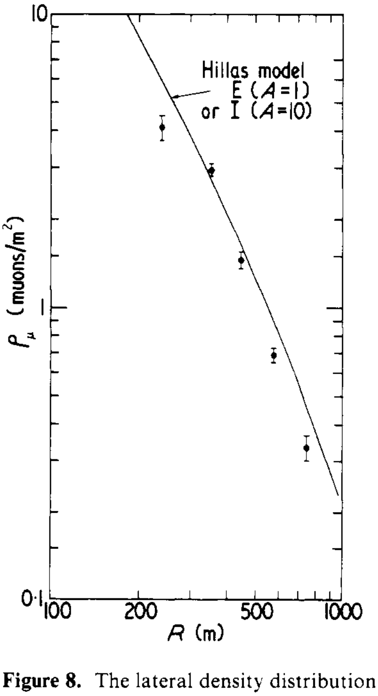


**Figure 8.** The lateral density distribution out to 1 km.

of mass $A = 10$. It will be seen that agreement with the experimental data is good.

A list of the absolute muon densities predicted by all the various Hillas' models at a core distance of 336 m is shown in table 4 for a shower size $\rho_{500} = 2{\cdot}66$ ve$\mu$m$^{-2}$.

**Table 4.** Absolute value of $\rho_u$ (336 m) predicted by Hillas' models ($\rho_{500} = 2{\cdot}66$ ve$\mu$m$^{-2}$, experimental value = $3{\cdot}0 \pm 0{\cdot}3$)

| Model | A | D | E | F | H | J | I | K |
|---|---|---|---|---|---|---|---|---|
| $A = 1$ | 3·44 | 1·90 | 3·12 | 3·74 | 3·79 | 3·40 | 2·82 | 2·28 |
| $A = 10$ | 3·74 | 2·27 | 3·45 | 3·95 | 3·97 | 3·60 | 3·15 | 2·59 |

## 7. Conclusions

The analysis of additional data obtained from the muon detector using stringent shower selection criteria has not revealed any gross disagreement with the analysis previously carried out. A structure function of the form first proposed by Linsley was found to give a good fit to the experimental points. The mean slope is in agreement with that obtained by the Sydney group. There is some evidence for a curving of the lateral distribution at core distances less than 200 m.

Comparison of the data with the predictions of Hillas tends to favour E and I. The best agreement with model E is for proton primaries and with model I for a mean primary mass number $A \sim 10$.

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